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Signals of New Physics using angular analysis in $B \to V_1 V_2$ decays*

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We show that an angular analysis of $B \to V_1 V_2$ decays yields numerous tests for new physics in the decay amplitudes. Many of these new-physics observables are nonzero even if the strong phase differences vanish. For certain observables, neither time-dependent measurements nor tagging is necessary. Should a signal for new physics be found, one can place a lower limit on the size of the new-physics parameters, as well as bound its effect on the measurement of the B^0–$\bar{B}^0$ mixing phase.

1 Introduction

CP violation in the B system is now firmly established. Successful runs at both Belle and BaBar detectors have made it possible for the weak phase β to be measured accurately [1]. This measurement of one of the interior angles of the unitarity triangle [2], is primarily performed using the so-called "gold-plated" mode $B_d^0(t) \to J/\psi K_s$. Having achieved this, attention is now being focused on measuring β using other modes. In the Standard model $B_d^0(t) \to \phi K_s$ and $B_d^0(t) \to \eta' K_s$ [3] also measure β to a good approximation. If the values of β measured using various modes were to disagree, it would provide an indication of New Physics (NP). Indeed, at present there appears to be a discrepancy between the value of β extracted from $B_d^0(t) \to J/\psi K_s$ and that obtained from $B_d^0(t) \to \phi K_s$ [4]. Should this difference remain as more data is accumulated, it would provide an indirect evidence for a NP amplitude in $B \to \phi K$. It is therefore important to explore other signals of NP, in order to corroborate this result.

Signals of NP obtained by comparing β extracted from $B_d^0(t) \to J/\psi K_s$ and $B_d^0(t) \to \phi K_s$, rely on the fact that the decay amplitude for $B_d^0(t) \to J/\psi K_s$ is dominated by a single contribution. In this case, the weak-phase information can be extracted cleanly, i.e. with no hadronic uncertainties [5]. However, this clean extraction is subject, to the absence of NP. If NP affects B_d^0–$\overline{B_d^0}$ mixing only, the analysis is unchanged, except that the measured value of β is not the true SM value, but rather one that has been shifted by a new-physics phase. On the other hand, if the NP affects the decay amplitude [7], then the extraction of β is no longer clean – it may be contaminated by hadronic uncertainties.

NP can affect the decay amplitude either at loop level (i.e. in the $b \to s$ penguin amplitude) or at tree level. Examples of such new-physics models include non-minimal supersymmetric models and models with Z-mediated flavor-changing neutral currents [8]. In all cases, if the new contributions have a different weak phase than that of the SM amplitude, then the measured value of β, β^{meas}, no longer corresponds to the phase of B_d^0–$\overline{B_d^0}$ mixing, β^{mix}. (Note that β^{mix} could include NP contributions to the mixing.)

If NP is present and contributes to the decay amplitude, it would be preferable to have *direct* evidence for this second amplitude. One would also like to obtain information about it (magnitude, weak and strong phases). It is therefore important to have as many independent tests as possible for NP. One possibility is to search for direct CP violation. However, direct CP asymmetries vanish if the strong phase difference between the SM and NP amplitudes is zero. It has been argued that this may well be the case in B decays, due to the fact that the b-quark is rather heavy. We show, however, that if one considers B-meson decays to two vector mesons, $B \to V_1 V_2$, many signals for NP emerge, including several that are nonzero even if the strong phase differences vanish. Furthermore, if *any* NP signal is found, one can place a lower bound on the size of the NP amplitude, and on the difference $|\beta^{meas} - \beta^{mix}|$. An angular analysis of any of the modes such as $B_d^0(t) \to J/\psi K^*$ or ϕK^*, $D^* D_s^*$ can be used for such a study. A similar analysis can be used within the SM to analyze decays such as $B_d^0(t) \to D^{*+} D^{*-}$.

In section 2 of the talk we examine how the large number of observables that $B \to V_1 V_2$, decay modes provide are modified in the presence of NP. In section 3 we derive '12' relations, the violation of any of which would signal NP. In section 4 we briefly discuss constraints on the size of NP as well as on $|\beta^{meas} - \beta^{mix}|$, which can be obtained if NP is observed.

2 Observables in $\mathbf{B \to V_1 V_2}$

Consider the decay $B \to V_1 V_2$, to which a single weak decay amplitude contributes within the SM. Suppose that there is a new-physics amplitude, with a different weak phase, that contributes to the decay. The decay amplitude

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for each of the three possible helicity states may be generally written as

$$\begin{aligned} A_\lambda \equiv Amp(B \to V_1 V_2)_\lambda &= a_\lambda e^{i\delta_\lambda^a} + b_\lambda e^{i\phi} e^{i\delta_\lambda^b} , \\ \bar{A}_\lambda \equiv Amp(\bar{B} \to \bar{V}_1 \bar{V}_2)_\lambda &= a_\lambda e^{i\delta_\lambda^a} + b_\lambda e^{-i\phi} e^{i\delta_\lambda^b} , \end{aligned} \tag{1}$$

where a_λ and b_λ represent the SM and NP amplitudes, respectively, ϕ is the new-physics weak phase, the $\delta_\lambda^{a,b}$ are the strong phases, and the helicity index λ takes the values $\{0, \|, \perp\}$. Using CPT invariance, the full decay amplitudes can be written as

$$\begin{aligned} \mathcal{A} &= Amp(B \to V_1 V_2) = A_0 g_0 + A_\| g_\| + i\, A_\perp g_\perp , \\ \bar{\mathcal{A}} &= Amp(\bar{B} \to \bar{V}_1 \bar{V}_2) = \bar{A}_0 g_0 + \bar{A}_\| g_\| - i\, \bar{A}_\perp g_\perp , \end{aligned} \tag{2}$$

where the g_λ are the coefficients of the helicity amplitudes written in the linear polarization basis. The g_λ depend only on the angles describing the kinematics [9]. The above equations enable us to write the time-dependent decay rates as

$$\begin{aligned} \Gamma(\overset{(-)}{B}_d(t) \to V_1 V_2) = e^{-\Gamma t} \sum_{\lambda \le \sigma} \Big(& \Lambda_{\lambda\sigma} \pm \Sigma_{\lambda\sigma} \cos(\Delta M t) \\ & \mp \rho_{\lambda\sigma} \sin(\Delta M t) \Big) g_\lambda g_\sigma . \end{aligned} \tag{3}$$

Thus, by performing a time-dependent angular analysis of the decay $B_d^0(t) \to V_1 V_2$, one can measure 18 observables. These are:

$$\begin{aligned} & \Lambda_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 + |\bar{A}_\lambda|^2), && \Sigma_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 - |\bar{A}_\lambda|^2), \\ & \Lambda_{\perp i} = -\mathrm{Im}(A_\perp A_i^* - \bar{A}_\perp \bar{A}_i^*), && \Lambda_{\|0} = \mathrm{Re}(A_\| A_0^* + \bar{A}_\| \bar{A}_0^*), \\ & \Sigma_{\perp i} = -\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*), && \Sigma_{\|0} = \mathrm{Re}(A_\| A_0^* - \bar{A}_\| \bar{A}_0^*), \\ & \rho_{\perp i} = \mathrm{Re}\Big(\frac{q}{p}\Big[A_\perp^* \bar{A}_i + A_i^* \bar{A}_\perp\Big]\Big), && \rho_{\perp\perp} = \mathrm{Im}\Big(\frac{q}{p}\, A_\perp^* \bar{A}_\perp\Big), \\ & \rho_{\|0} = -\mathrm{Im}\Big(\frac{q}{p}[A_\|^* \bar{A}_0 + A_0^* \bar{A}_\|]\Big), && \rho_{ii} = -\mathrm{Im}\Big(\frac{q}{p} A_i^* \bar{A}_i\Big), \end{aligned} \tag{4}$$

where $i = \{0, \|\}$. In the above, $q/p = \exp(-2\, i\beta^{mix})$, where β^{mix} is the weak phase describing B_d^0–$\bar{B}_d^0$ mixing. Note that β^{mix} may include NP effects (in the SM, $\beta^{mix} = \beta$). Note also that the signs of the various ρ terms depend on the CP-parity of the various helicity states. We have chosen the sign of ρ_{00} and $\rho_{\|\|}$ to be -1, which corresponds to the final state $J/\psi K^*$.

The 18 observables given above can be written in terms of 13 theoretical parameters: three a_λ's, three b_λ's, β^{mix}, ϕ, and five strong phase differences defined by $\delta_\lambda \equiv \delta_\lambda^b - \delta_\lambda^a$, $\Delta_i \equiv \delta_\perp^a - \delta_i^a$. The explicit expressions for the observables are as follows:

$$\begin{aligned} \Lambda_{\lambda\lambda} &= a_\lambda^2 + b_\lambda^2 + 2 a_\lambda b_\lambda \cos\delta_\lambda \cos\phi , \\ \Sigma_{\lambda\lambda} &= -2 a_\lambda b_\lambda \sin\delta_\lambda \sin\phi , \\ \Lambda_{\perp i} &= 2\, [a_\perp b_i \cos(\Delta_i - \delta_i) - a_i b_\perp \cos(\Delta_i + \delta_\perp)] \sin\phi , \\ \Lambda_{\|0} &= 2\Big[a_\| a_0 \cos(\Delta_0 - \Delta_\|) + a_\| b_0 \cos(\Delta_0 - \Delta_\| - \delta_0) \cos\phi \\ &\quad + a_0 b_\| \cos(\Delta_0 - \Delta_\| + \delta_\|) \cos\phi \\ &\quad + b_\| b_0 \cos(\Delta_0 - \Delta_\| + \delta_\| - \delta_0)\Big] , \\ \Sigma_{\perp i} &= -2\Big[a_\perp a_i \sin\Delta_i + a_\perp b_i \sin(\Delta_i - \delta_i)\cos\phi \\ &\quad + a_i b_\perp \sin(\Delta_i + \delta_\perp)\cos\phi + b_\perp b_i \sin(\Delta_i + \delta_\perp - \delta_i)\Big] , \\ \Sigma_{\|0} &= 2\Big[a_\| b_0 \sin(\Delta_0 - \Delta_\| - \delta_0) \\ &\quad - a_0 b_\| \sin(\Delta_0 - \Delta_\| + \delta_\|)\Big] \sin\phi , \\ \rho_{ii} &= a_i^2 \sin 2\beta^{mix} + b_i^2 \sin(2\beta^{mix} + 2\phi) \\ &\quad + 2 a_i b_i \cos\delta_i \sin(2\beta^{mix} + \phi) , \\ \rho_{\perp\perp} &= -a_\perp^2 \sin 2\beta^{mix} - b_\perp^2 \sin(2\beta^{mix} + 2\phi) \\ &\quad - 2 a_\perp b_\perp \cos\delta_\perp \sin(2\beta^{mix} + \phi) , \\ \rho_{\perp i} &= 2\Big[a_i a_\perp \cos\Delta_i \cos 2\beta^{mix} \\ &\quad + a_\perp b_i \cos(\Delta_i - \delta_i)\cos(2\beta^{mix} + \phi) \\ &\quad + a_i b_\perp \cos(\Delta_i + \delta_\perp)\cos(2\beta^{mix} + \phi) \\ &\quad + b_i b_\perp \cos(\Delta_i + \delta_\perp - \delta_i)\cos(2\beta^{mix} + 2\phi)\Big] , \\ \rho_{\|0} &= 2\Big[a_0 a_\| \cos(\Delta_0 - \Delta_\|) \sin 2\beta^{mix} \\ &\quad + a_\| b_0 \cos(\Delta_0 - \Delta_\| - \delta_0) \sin(2\beta^{mix} + \phi) \\ &\quad + a_0 b_\| \cos(\Delta_0 - \Delta_\| + \delta_\|) \sin(2\beta^{mix} + \phi) \\ &\quad + b_0 b_\| \cos(\Delta_0 - \Delta_\| + \delta_\| - \delta_0) \sin(2\beta^{mix} + 2\phi)\Big] . \end{aligned} \tag{5}$$

It is straightforward to show that one cannot extract β^{mix}. There are a total of six amplitudes describing $B \to V_1 V_2$ and $\bar{B} \to \bar{V}_1 \bar{V}_2$) decays [Eq. (1)]. Thus, at best one can measure the magnitudes and relative phases of these six amplitudes, giving 11 measurements. Since the number of measurements (11) is fewer than the number of theoretical parameters (13), one cannot obtain any of the theoretical unknowns purely in terms of observables. In particular, it is impossible to extract β^{mix} cleanly.

3 Signals of New Physics

In the absence of NP, $b_\lambda = 0$. The number of parameters is then reduced from 13 to 6: three a_λ's, two strong phase differences (Δ_i), and β^{mix}. All of these can be determined cleanly in terms of observables. There are 18 observables, but only 6 theoretical parameters, thus 12 relations must exist among the observables in the absence of NP. (Of course, only five of these are independent.) These 12 relations are:

$$\begin{aligned} & \Sigma_{\lambda\lambda} = \Lambda_{\perp i} = \Sigma_{\|0} = 0 \\ & \frac{\rho_{ii}}{\Lambda_{ii}} = -\frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}} = \frac{\rho_{\|0}}{\Lambda_{\|0}} \\ & \Lambda_{\|0} = \frac{1}{2\Lambda_{\perp\perp}}\Big[\frac{\Lambda_{\lambda\lambda}^2 \rho_{\perp 0}\rho_{\perp\|} + \Sigma_{\perp 0}\Sigma_{\perp\|}(\Lambda_{\lambda\lambda}^2 - \rho_{\lambda\lambda}^2)}{\Lambda_{\lambda\lambda}^2 - \rho_{\lambda\lambda}^2}\Big] \end{aligned}$$

$$\frac{\rho_{\perp i}^2}{4\Lambda_{\perp\perp}\Lambda_{ii} - \Sigma_{\perp i}^2} = \frac{\Lambda_{\perp\perp}^2 - \rho_{\perp\perp}^2}{\Lambda_{\perp\perp}^2} \,. \tag{6}$$

The important consequence is [10] that *the violation of any of the above relations will be a smoking-gun signal of NP.* It may be emphasized that the angular analysis of $B \to V_1 V_2$ decays provides numerous tests for the presence of NP.

The observable $\Lambda_{\perp i}$ deserves special attention [11]. From Eq. (5), one sees that even if the strong phase differences vanish, $\Lambda_{\perp i}$ is nonzero in the presence of NP ($\phi \neq 0$), in stark contrast to the direct CP asymmetries (proportional to $\Sigma_{\lambda\lambda}$). This is due to the fact that the $\perp$ helicity is CP-odd, while the 0 and $\|$ helicities are CP-even. While the reconstruction of the full $B_d^0(t)$ and $\overline{B_d^0}(t)$ decay rates in Eq. (3) requires both tagging and time-dependent measurements, the $\Lambda_{\lambda\sigma}$ terms survive even if the two rates for $B_d^0(t)$ and $\overline{B_d^0}(t)$ decays are added together. We note also that these terms are time-independent. Therefore, *no tagging or time-dependent measurements are needed to extract* $\Lambda_{\perp i}$! It is only necessary to perform an angular analysis of the final state $V_1 V_2$. Thus, this measurement can even be made at a symmetric B-factory. The decays of charged B mesons to vector-vector final states are even simpler to analyze since no mixing is involved. One can in principle combine charged and neutral B decays to increase the sensitivity to NP. A nonzero value of $\Lambda_{\perp i}$ would provide a clear signal for NP [12].

The decays of both charged and neutral B mesons to $D^* D_s^*$ can be analyzed similarly. Because these modes are dominated by a single decay amplitude in the SM, no direct CP violation is expected. Further, since this is not a final state to which both B_d^0 and $\overline{B_d^0}$ can decay, no indirect CP violation is possible either. An angular analysis of these decays would therefore be very interesting in exploring the presence of NP.

It must be noted that, despite the large number of new-physics signals, it is still possible for the NP to remain hidden. This happens if a singular situation is realized. If the three strong phase differences δ_λ vanish, and the ratio $r_\lambda \equiv b_\lambda / a_\lambda$ is the same for all helicities, i.e. $r_0 = r_\| = r_\perp$, then it is easy to show that the relations in Eq. (6) are all satisfied. Thus, if the NP happens to respect these very special conditions, the angular analysis of $B \to V_1 V_2$ would show no signal for NP, yet the measured value of β would not correspond to the actual phase of B_d^0–$\overline{B_d^0}$ mixing.

4 Constraints on the size of New Physics

We have argued earlier, that in the presence of NP one cannot extract the true value of β^{mix}. However, as we will describe below, the angular analysis does allow one to constrain the value of the difference $|\beta^{meas} - \beta^{mix}|$, as well as the size of the NP amplitudes b_λ^2. Naively, it appears impossible to obtain any constraints on the NP parameters, since we have 11 measurements, but 13 theoretical unknown parameters. However, because the equations are nonlinear, such constraints are possible. Below, we list some of these constraints [10]

In the constraints, we will make use of the following quantities. For the vector-vector final state, the analogue of the usual direct CP asymmetry is $a_\lambda^{dir} \equiv \Sigma_{\lambda\lambda}/\Lambda_{\lambda\lambda}$, which is helicity-dependent. For convenience, we define the related quantity $y_\lambda^2 = (1 - \Sigma_{\lambda\lambda}^2/\Lambda_{\lambda\lambda}^2)$. The measured value of $\sin 2\beta$ can also depend on the helicity of the final state: $\rho_{\lambda\lambda}$ can be recast in terms of a measured weak phase $2\beta_\lambda^{meas}$, defined as $\sin 2\beta_\lambda^{meas} = \pm\rho_{\lambda\lambda}/(\Lambda_{\lambda\lambda} y_\lambda)$, where the + (−) sign corresponds to $\lambda = 0, \|$ ($\perp$). In terms of these quantities, the size of NP amplitudes b_λ^2 may be expressed as

$$2\, b_\lambda^2 \sin^2\phi = \Lambda_{\lambda\lambda}\Big(1 - y_\lambda \cos(2\beta_\lambda^{meas} - 2\beta)\Big) \,. \tag{7}$$

The form of the constraints depends on which new-physics signals are observed; we give a partial list below. For example, suppose that direct CP violation is observed in a particular helicity state. In this case a lower bound on the corresponding NP amplitude can be obtained by minimizing b_λ^2 with respect to β and ϕ:

$$b_\lambda^2 \geq \frac{1}{2}\Lambda_{\lambda\lambda}\left[1 - y_\lambda\right] . \tag{8}$$

On the other hand, suppose that the new-physics signal is $\beta_i^{meas} \neq \beta_j^{meas}$. Defining $2\omega \equiv 2\beta_j^{meas} - 2\beta_i^{meas}$ and $\eta_\lambda \equiv 2(\beta_\lambda^{meas} - \beta^{mix})$, the minimization of $(b_i^2 \mp b_j^2)$ with respect to η_i and ϕ yields

$$(b_i^2 \mp b_j^2) \geq \frac{\Lambda_{ii} \mp \Lambda_{jj}}{2} - \frac{\left|y_i \Lambda_{ii} \mp y_j \Lambda_{jj} e^{2i\omega}\right|}{2} \,, \tag{9}$$

where $\Lambda_{ii} > \Lambda_{jj}$ is assumed. If there is no direct CP violation ($\Sigma_{\lambda\lambda} = 0$), but $\Lambda_{\perp i}$ is nonzero, one has

$$2(b_i^2 \mp b_\perp^2) \geq \Lambda_{ii} \mp \Lambda_{\perp\perp} - \sqrt{(\Lambda_{ii} \mp \Lambda_{\perp\perp})^2 \pm \Lambda_{\perp i}^2} \,. \tag{10}$$

One can also obtain bounds on $|\beta_\lambda^{meas} - \beta^{mix}|$, though this requires the nonzero measurement of observables involving the interference of different helicities. For example, if $\Lambda_{\perp i}$ is nonzero and $\Sigma_{\lambda\lambda} = 0$, we find

$$\Lambda_{ii}\cos\eta_i + \Lambda_{\perp\perp}\cos(\eta_\perp - 2\eta_i) \leq \sqrt{(\Lambda_{ii} + \Lambda_{\perp\perp})^2 - \Lambda_{\perp i}^2} \,,$$

$$\Lambda_{ii}\cos\eta_i - \Lambda_{\perp\perp}\cos\eta_\perp \leq \sqrt{(\Lambda_{ii} - \Lambda_{\perp\perp})^2 + \Lambda_{\perp i}^2} \,. \tag{11}$$

If $\Lambda_{\perp i} \neq 0$, one cannot have $\eta_i = \eta_\perp = 0$. These constraints therefore place a lower bound on $|\beta_i^{meas} - \beta^{mix}|$ and/or $|\beta_\perp^{meas} - \beta^{mix}|$.

A-priori, one does not know which of the above constraints is the strongest – this depends on the actual values of the observables. Of course, in practice, one will simply perform a fit to obtain the best lower bounds on these NP parameters [10]. However, it is interesting to study analytically the dependence of constraints as a function of observables which would signal NP if non-zero.

If the apparent discrepancy in the value of $\sin 2\beta$ as obtained from measurements of $B_d^0(t) \to J/\psi K_s$ and $B_d^0(t) \to \phi K_s$ [4] is on account of NP, angular analyses of $B_d^0(t) \to J/\psi K^*$ and $B_d^0(t) \to \phi K^*$ would allow one to determine if NP is indeed present. If NP signal is confirmed, this analysis would allow one to put constraints on the NP parameters.

Finally, we note that this analysis can also be applied within the SM to decays such as $B_d^0(t) \to D^{*+}D^{*-}$. These decays have both a tree and a penguin contribution, so that β^{mix} cannot be extracted cleanly. Assuming no NP, the above analysis allows one to obtain lower bounds on the ratio of penguin to tree amplitudes, as well as on $|\beta_\lambda^{meas} - \beta^{mix}|$. This can serve as a cross-check on other measurements of β^{mix}, as well as on model calculations of the hadronic amplitudes.

5 Summary

In the standard model (SM), the cleanest extraction of the CP angles comes from neutral B decays that are dominated by a single decay amplitude. If there happens to be a new-physics (NP) contribution to the decay amplitude, with a different weak phase, this could seriously affect the cleanliness of the measurement. There is already a hint of such NP, as indicated by the discrepancy between the value of β extracted from $B_d^0(t) \to J/\psi K_s$ and that obtained from $B_d^0(t) \to \phi K_s$. However, it is important to confirm this through independent direct tests, and to make an attempt to obtain information about the NP amplitude, if possible.

We have shown that this type of NP can be probed by performing an angular analysis of the related $B \to V_1 V_2$ decay modes. There are numerous relations that are violated in the presence of NP, and several of these signals remain nonzero even if the strong phase difference between the SM and NP amplitudes vanishes. The most incisive test is a measurement of $\Lambda_{\perp i} \neq 0$. To obtain this observable, neither tagging nor time-dependent measurements are necessary – one can combine all neutral and charged B decays.

Furthermore, should a signal for NP be found, one can place a lower bound on the difference $|\beta^{meas} - \beta^{mix}|$, as well as on the size of the NP amplitudes. By applying this analysis to the decays $B_d^0(t) \to J/\psi K^*$ and $B_d^0(t) \to \phi K^*$, one can confirm the presence of the NP that is hinted at in the measurements of $B_d^0(t) \to J/\psi K_s$ and $B_d^0(t) \to \phi K_s$ [4]. It can even be applied within the SM to analyze decays such as $B_d^0(t) \to D^{*+}D^{*-}$, which receive both tree and penguin contributions.

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References

1. B. Aubert et al., BABAR Collaboration, Phys. Rev. Lett. **89**: 201802 (2002); K. Abe et al., BELLE Collaboration, *Phys. Rev.* **D66**: 071102 (192002); G. Raven, these proceedings.
2. For a review, see, for example, *The BaBar Physics Book*, eds. P.F. Harrison and H.R. Quinn, SLAC Report 504, 1998.
3. D. London and A. Soni, *Phys. Lett.* **407B**, 61 (1997).
4. For $B_d^0(t) \to J/\psi K_s$, the world average is $\sin 2\beta = 0.734 \pm 0.054$, whereas for $B_d^0(t) \to \phi K_s$ it is $\sin 2\beta = -0.39 \pm 0.41$. See Ref.[1] and K. Abe et al., BELLE Collaboration, hep-ex/0212062; B. Aubert et al., BABAR Collaboration, hep-ex/0207070.
5. In fact, there are two weak amplitudes that can contribute even to $B_d^0(t) \to J/\psi K_s$: the tree amplitude and the $b \to s$ penguin amplitude. However, the weak phases of these two amplitudes are equal (they vanish in the Wolfenstein parametrization of the CKM matrix [6]), so that there is effectively only a single weak amplitude contributing to $B_d^0 \to J/\psi K_s$. Thus, the extraction of the CP phase β from this decay mode is extremely clean.
6. L. Wolfenstein, *Phys. Rev. Lett.* **51**, 1945 (1983).
7. Y. Grossman and M.P. Worah, *Phys. Lett.* **395B**, 241 (1997).
8. See, for example, M. Gronau and D. London, *Phys. Rev.* **D55**, 2845 (1997); See also Ref. [7].
9. N. Sinha and R. Sinha, *Phys. Rev. Lett.* **80**, 3706 (1998); A.S. Dighe, I. Dunietz and R. Fleischer, *Eur. Phys. J.* **C6**, 647 (1999).
10. D. London, N. Sinha and R. Sinha, hep-ph/0304230; D. London, N. Sinha and R. Sinha, in preparation.
11. The observables $\Lambda_{\perp i}$, which are also triple-product correlations, have also been studied in $B \to V_1 V_2$ decays in A. Datta and D. London, hep-ph/0303159.
12. This point has also been made in X.-G. He and W.-S. Hou, *Phys. Rev.* **D58**: 117502 (1998); D. London, N. Sinha and R. Sinha, hep-ph/0207007, published in the proceedings of the First International Conference on Flavor Physics and CP Violation (FPCP 2002), edited by R.G.C. Oldeman, eConf C020516 (2002).